## Holographic Teleportation in Higher Dimensions

Byoungjoon Ahn (GIST) collaborated with Yongjun Ahn, Sang-Eon Bak, Victor Jahnke, and Keun-Young Kim

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# Motivation

## Wormholes & Averaged Null Energy Condition

• Asymptotically AdS two-sided black hole

 $\leftrightarrow$  two copies of the CFT entangled in a TFD state

• Wormhole connecting the two regions is not traversable



• The Averaged Null Energy Condition(ANEC)

: The stress energy tensor integrated along a complete achronal null geodesic is always non-negative

$$\int T_{\mu\nu}k^{\mu}k^{\nu}d\lambda \geq 0$$

#### **Traversable Wormholes**

• ANEC can be violated if one introduces a double trace deformation that couples the two boundary theories.[Gao, Jafferis, and Wall, 16']

$$\delta S = \int dt dx h(t, x) \mathcal{O}_L(-t, x) \mathcal{O}_R(t, x)$$

• In the semiclassical approximation, one can write the Einstein's equation as

$$G_{\mu
u} = 8\pi G_N \left\langle T_{\mu
u} \right
angle$$

The deformation can be chosen in such a way that makes the expectation value of the stress energy tensor violate the ANEC.



• We work in the context of Einstein gravity

$$S = \frac{1}{16\pi G_N} \int d^{d+1}x \left[ R + \frac{d(d-1)}{\ell^2} \right]$$
$$ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{r^2 - 1} + r^2(d\chi^2 + \sinh^2\chi d\Omega_{d-2}^2)$$

 $\ell$  denotes the AdS radius and set  $\ell=1.$ 

• It has a non-zero Hawking temperature given by  $T = \frac{1}{\beta} = \frac{1}{2\pi}$ 



• In terms of Kruskal-Szekeres coordinates, the metric becomes

$$ds^2 = -rac{4dU\,dV}{(1+UV)^2} + \left(rac{1-UV}{1+UV}
ight)^2 d\mathbb{H}_{d-1}^2$$

where  $d\mathbb{H}_{d-1}^2 = d\chi^2 + \sinh^2 \chi d\Omega_{d-2}^2$  and  $x \in d\mathbb{H}_{d-1}^2$ .

- The AdS boundary : UV = -1
- Two horizons : U = 0 and V = 0



• The linearized Einstein's equation in Kruskal coordinates

$$\frac{d-1}{2\ell^2}\left(h_{UU}+\frac{\partial_U(Uh_{UU})}{\partial_U(Uh_{UU})}\right)+\frac{1}{2\ell^2}\partial_U^2h_{\chi\chi}=8\pi G_N T_{UU}$$

- A null ray (the past infinity  $\rightarrow$  future infinity) along the horizon undergoes a shift in the V direction

$$\Delta V = -\frac{1}{2g_{UV}(0)} \int_{-\infty}^{\infty} dU h_{UU} = \frac{4\pi G_N}{d-1} \int T_{UU} dU$$

 $\Delta V$  becomes negative if  $\int {\cal T}_{UU} dU < 0$ 



# ANEC Violation - Point Splitting Method

• We consider a double-trace deformation of the form given by

$$\delta H(t) = \int d\mathsf{x} h(t,\mathsf{x}) \mathcal{O}_L(-t,\mathsf{x}) \mathcal{O}_R(t,\mathsf{x})$$

and we take  $h(t, x) = h \theta(t - t_0)$ .

• Scaling dimension of scalar operator

$$\Delta_{\mathcal{O}} = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2} \quad \text{orm} \neq -\left(\frac{d}{2}\right)^2$$

$$\psi(r \to \infty_R) \to \alpha_R(t, \mathbf{x})r^{-\Delta} + \beta_R(t, \mathbf{x})r^{-d+\Delta}$$
$$\beta_R(t, \mathbf{x}) = h(t, \mathbf{x})\alpha_L(-t, \mathbf{x})$$

- Condition for  $\delta H$ 
  - relevant deformation :  $\Delta_{\mathcal{O}} \leq d/2$
  - unitary bound :  $d/2 1 \leq \Delta_\mathcal{O}$

#### **Point Splitting Method1**

• In the context of a semiclassical approximation,

$$G_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu} \rangle$$

• For a scalar field with an action

$$S_{
m scalar} = -rac{1}{2}\int d^{d+1}x\sqrt{-g}(g^{\mu
u}\partial_{\mu}\phi\partial_{
u}\phi+m^2\phi^2)$$

the 1-loop expectation value of the stress energy tensor

$$\langle T_{\mu\nu} \rangle = \lim_{x \to x'} \partial_{\mu} \partial'_{\nu} G(x, x') - \frac{1}{2} g_{\nu} g^{\alpha\beta} \partial_{\alpha} \partial'_{\beta} G(x, x') - \frac{1}{2} m^2 g_{\mu\nu} G(x, x')$$

• Renormalized scalar two point function under the presence of the deformation

$$\begin{aligned} G(x,x') = & \langle \phi_R^H(t,r,x) \phi_R^H(t',r',x') \rangle \\ = & \langle U^{-1}(t,t_0) \phi_R'(t,x) U(t,t_0) U^{-1}(t',t_0) \phi_R'(t',x') U(t',t_0) \rangle \end{aligned}$$

where  $U(t, t_0) = \mathcal{T} e^{-i \int_{t_0}^t dt \delta H(t)}$ .

• By considering a small h expansion,

$$G(x, x') = G_0(x, x') + G_1(x, x')h + O(h^2)$$

• The 1-loop contribution evaluating at V = 0 is given by

$$\begin{aligned} G_1(U, U', \mathbf{x}, \mathbf{x}') &= 2\sin\left(\pi\Delta\right) c_\Delta^2 \int \frac{dU_1}{U_1} \int d\mathbf{x}_1 h(U_1, \mathbf{x}_1) \,\theta\!\left(\frac{U}{U_1} - \cosh d(\mathbf{x}, \mathbf{x}_1)\right) \\ &\times \!\left(\frac{1}{\frac{U}{U_1}} - \cosh d(\mathbf{x}, \mathbf{x}_1)\right)^\Delta\!\left(\frac{1}{U'U_1 + \cosh d(\mathbf{x}', \mathbf{x}_1)}\right)^\Delta + (U, \mathbf{x} \leftrightarrow U', \mathbf{x}') \\ &\equiv F(U, U', \mathbf{x}, \mathbf{x}') + F(U', U, \mathbf{x}, \mathbf{x}'), \end{aligned}$$

where  $d(x, x_1)$  is the geodesic distance between x and  $x_1$  in  $\mathbb{H}_{d-1}$ .

• For simplicity, we set x = x' = 0

$$T_{UU} = 2 \lim_{U' \to U} \partial_U \partial_{U'} F(U, U')$$

By considering a small h expansion up to linear order,

$$\int_{U_0}^{\infty} dU T_{UU} = -\frac{h\pi^{-\frac{d}{2}} \Gamma\left(\Delta + \frac{1}{2}\right) \Gamma\left(\Delta + \frac{3-d}{2}\right)}{(\Delta + \frac{1}{2}) \Gamma\left(\Delta + \frac{2-d}{2}\right)^2} \frac{{}^2 F_1\left(\Delta + \frac{1}{2}, \frac{1}{2} - \Delta, \Delta + \frac{3}{2}; \frac{1}{1+U_0^2}\right)}{(1+U_0^2)^{\Delta + \frac{1}{2}}}$$



## **ANEC Violation - Eikonal Approximation**

We use the eikonal approximation to directly compute the expectation value of a two-sided correlation function. [Maldacena, Stanford, Yang, 17']

$$\langle [\psi_L, \psi_R] \rangle_{\mathcal{V}} \equiv \left\langle \left[ \psi_L(-t_1, \mathsf{x}_1), e^{-i\mathcal{V}} \psi_R(t_2, \mathsf{x}_2) e^{i\mathcal{V}} \right] \right\rangle$$

Double trace deformation involving K light fields

$$\mathcal{V} = \frac{1}{K} \sum_{i=1}^{K} \int dt' \, dx' h(t', \mathbf{x}') \mathcal{O}_{L}^{i}(-t', \mathbf{x}') \mathcal{O}_{R}^{i}(t', \mathbf{x}')$$

The leading response of  $\psi_L$  to a unitary perturbation to the R system by  $e^{i\epsilon\psi_R}$ 

$$\langle e^{-\imath\epsilon\psi_R}e^{-\imath g\mathcal{V}}\psi_L e^{\imath g\mathcal{V}}e^{\imath\epsilon\psi_R} \rangle = \langle e^{-\imath g\mathcal{V}}\psi_L e^{\imath g\mathcal{V}} \rangle - \imath\epsilon\langle [\psi_L,\psi_R] \rangle_{\mathcal{V}} + \mathcal{O}(g^2)$$

## **Eikonal Approximation Method 2**

Introducing

$$C = \langle e^{-i\mathcal{V}}\psi_R(t_2, \mathsf{x}_2)e^{i\mathcal{V}}\psi_L(-t_1, \mathsf{x}_1)\rangle$$

The imaginary part gives the original commutator.

$$\langle \left[\psi_R, e^{-ig\mathcal{V}}\psi_L e^{i\mathcal{V}}\right] \rangle = -2\mathrm{Im}(C)$$

In the large K and small  $G_N$  limits,

$$\mathcal{C}=e^{-i\langle\mathcal{V}
angle} ilde{\mathcal{C}}\,,\;\; ilde{\mathcal{C}}\equiv\langle\psi_{\mathcal{R}}(t_2,\mathsf{x}_2)e^{i\mathcal{V}}\psi_{\mathcal{L}}(-t_1,\mathsf{x}_1)
angle$$



Consider a small h expansion at linear order and set K = 1

$$\tilde{C}_1 = i \int dt' dx' \langle \psi_R(t_2, \mathsf{x}_2) \mathcal{O}_L(-t', \mathsf{x}') \mathcal{O}_R(t', \mathsf{x}') \psi_L(-t_1, \mathsf{x}_1) \rangle h(t', \mathsf{x}')$$

which is basically an out-of-time-order correlator.

We first write  $\tilde{\mathcal{C}}_1$  as an amplitude

$$\begin{split} \tilde{\mathcal{C}}_1 &= i \left\langle \mathsf{out} | \mathsf{in} \right\rangle \\ &|\mathsf{in} \rangle = |\mathcal{O}_R(t',\mathsf{x}')\psi_L(-t_1,\mathsf{x}_1)\rangle \\ &|\mathsf{out} \rangle = |\mathcal{O}_L^{\dagger}(-t',\mathsf{x}')\psi_R^{\dagger}(t_2,\mathsf{x}_2)\rangle \end{split}$$

and the phase shift  $\delta$  is given by  $\textit{S}_{\textit{classical}}$  in the shock wave geometry

$$\delta = 16\pi G_N q^U p^V f(\tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_4)$$

where  $f(\tilde{x}_1, \tilde{x}_4)$  is a transverse profile.

$$\begin{split} \tilde{\mathcal{C}}_{1} &= \alpha^{2} \int dq^{U} d\tilde{\mathbf{x}}_{1} \left[ q^{U} \Psi_{\psi_{R}}^{*}(q^{U}, \tilde{\mathbf{x}}_{1}) \Psi_{\psi_{L}}(q^{U}, \tilde{\mathbf{x}}_{1}) \right] \\ &\times \int dp^{V} d\tilde{\mathbf{x}}_{4} \int dt' d\mathsf{x}' \left[ h(t', \mathsf{x}') p^{V} \Psi_{\mathcal{O}_{L}}^{*}(p^{V}, \tilde{\mathbf{x}}_{4}) \Psi_{\mathcal{O}_{R}}(p^{V}, \tilde{\mathbf{x}}_{4}) e^{i\delta} \right] \end{split}$$

At all orders in h,

$$ilde{C} = lpha \int dq \, d ilde{x}_1 q \, \Psi^*_{\psi_R}(q, ilde{x}_1) \Psi_{\psi_L}(q, ilde{x}_1) \, e^{-iD}$$

where

$$D = -\alpha \int dp d\tilde{x}_4 \int dt' dx' h(t', x') p \Psi^*_{\mathcal{O}_L}(p, \tilde{x}_4) \Psi_{\mathcal{O}_R}(p, \tilde{x}_4) e^{i\delta}$$

where we replace  $p_1^U$  and  $p_4^V$  by q and p, respectively.

#### **Probe Approximation**

- $\Phi_{\psi}$  : a positive-energy shock wave  $\rightarrow$  close the window of the wormhole
- Probe limit: the backreaction of the signal is too small to deform the perturbed geometry by the negative-energy shock wave.
- Expansion to the first order in q, we find  $D = D_0 + D_1 q$ .

$$D_{1} = -\alpha g_{N} b_{\mathcal{O}}^{2} \frac{\Delta_{\mathcal{O}} \Gamma(2\Delta_{\mathcal{O}})}{2} \int dt' d\mathsf{x}' d\tilde{\mathsf{x}}_{4} \frac{h(t',\mathsf{x}') f(\tilde{\mathsf{x}}_{1},\tilde{\mathsf{x}}_{4})}{\left[\cosh t' \cosh d(\tilde{\mathsf{x}}_{4},\mathsf{x}')\right]^{2\Delta_{\mathcal{O}}+1}}$$

• The zeroth order term  $D_0$  cancels the overall factor of  $e^{-i\langle V \rangle}$  in the correlator  $C = e^{-i\langle V \rangle} \tilde{C}$ .

$$C_{\text{probe}} = \langle \psi_R e^{-i\Delta_1 q} \psi_L \rangle = \langle \psi_R e^{-i\Delta V \hat{\rho}_V} \psi_L \rangle$$

The ANE can be computed as

$$\int T_{UU} dU = \frac{d-1}{4\pi G_N} \Delta V = \frac{d-1}{4\pi G_N} D_{U}$$

- Homogeneous perturbation : the boundary operators independent on the coordinates x ∈ ℍ<sub>d-1</sub>
- The signal produced by a local operator
- We consider the perturbation :  $h(t', x') = h \delta(t' t_0)$

#### **Classical Action & Transverse Profile**

• The stress energy tensors of the shock waves :

$$T_{UU}^{-} = \frac{q_U}{r_0^{d-1}} \delta(U), \qquad T_{VV}^{+} = \frac{p_V}{r_0^{d-1}} \delta(V)$$

where  $r_0$  is the horizon radius and  $q_U \& p_V$  are the total momentums.

• The corresponding backreaction on the geometry :

$$ds^2 \to ds^2 + h_{UU}^- dU^2$$
,  $h_{UU}^- = \frac{16\pi G_N}{r_0^{d-3}} q_U \,\delta(U) \, f(\mathbf{x} - \mathbf{x}')$ 

• The shock wave transverse profile satisfies the following equation :

$$\left(\Box_{\mathbb{H}_{d-1}} - \frac{2\pi}{\beta}r_0(d-1)\right)f(\mathsf{x}) = 1$$

Finally we can obtain:

$$h_{UU}^{-} = rac{16\pi G_N}{r_0^{d-3}} q_U \, \delta(U) \, rac{1}{\mu} \, , \ \ \mu \equiv (d-1) r_0^2$$

The phase shift of the collision between the two shocks

$$\delta = S_{\text{classical}} = \frac{1}{2} \int d^{d+1} x \sqrt{-g} h_{UU}^{-} T_{+}^{UU} = \frac{4\pi G_N}{r_0^{d-1}} \frac{q_U p_V^{\text{tot}}}{d-1}$$

#### • Negative shift

$$D_1 = -\alpha b_{\mathcal{O}}^2 \frac{\Delta_{\mathcal{O}} \Gamma(2\Delta_{\mathcal{O}})}{2(d-1)} \int dt' dx' \frac{16\pi G_N h(t',x')}{\left[\cosh t' \cosh d(x',0)\right]^{2\Delta_{\mathcal{O}}+1}}$$

• Averaged null energy for the instantaneous source

$$\begin{aligned} \mathcal{A}^{\text{inst}}(U) &= \frac{1}{\text{vol}(S_{d-2})} \int T_{UU} dU \\ &= -h\pi^{\frac{1}{2}-d} \frac{\Gamma(\frac{d-1}{2})\Gamma(\Delta_{\mathcal{O}} + \frac{1}{2})\Gamma(\Delta_{\mathcal{O}} + \frac{3-d}{2})}{\Gamma(\Delta_{\mathcal{O}} - \frac{d-2}{2})^2} \left(\frac{U_0}{1 + U_0^2}\right)^{2\Delta_{\mathcal{O}} + 1} \end{aligned}$$

where  $U_0 = e^{t_0}$ .

### **Result2 - Homogeneous Shock**

• 
$$C_{\text{probe}} = \langle \psi_R e^{-iD_1 q} \psi_L \rangle$$
  
 $= -\alpha 2^{4\Delta_{\psi}} b_{\psi}^2 \int d\tilde{x}_1 \frac{\Gamma(2\Delta_{\psi}) e^{(t_1 - t_2)\Delta_{\psi}}}{\left[2\left(e^{-t_2}\cosh d(\tilde{x}_1, x_2) + e^{t_1}\cosh d(\tilde{x}_1, x_1)\right) - D_1\right]^{2\Delta_{\psi}}}$   
•  $\chi_1 = -\chi_2 = \frac{\Delta_{\chi}}{2} \rightarrow \text{the correlator depends on } (t1, t2, \Delta_{\chi}), \Delta_{\psi} \& D_1$ 

•  $T_c \sim \log D_1$  is a time scale related to scrambling time



• Deformation and signal are produced by local operators

• 
$$h(t', x') = h \, \delta(t' - t_0) \delta(x', 0)$$

• Transverse profile :  $f(x) = \frac{1}{d}e^{-(d-1)x}$ 

$$C_{\text{probe}} \sim -\int d\tilde{x}_1 \frac{\Gamma(2\Delta_{\psi})e^{(t_1-t_2)\Delta_{\psi}}}{\left[2\left(e^{-t_2}\cosh d(\tilde{x}_1, x_2) + e^{t_1}\cosh d(\tilde{x}_1, x_1)\right) - D_1(\tilde{x}_1)\right]^{2\Delta_{\psi}}}$$

•  $D_1$  maintains a dependence on  $\tilde{x}_1$  in higher dimensional cases

#### **Result3 - Localized Shock**



#### Sweet Spot - Optimal Point

• Saddle point approximation :  $1 \ll \Delta_{\psi} \ll \Delta_{\mathcal{O}}$ 

$$C_{
m probe} \sim rac{1}{\left[1-e^{ au}- au^*-(d-1)X
ight]^{2\Delta_\psi}}$$

- Chaos-related properties of the boundary theory are encoded
- Lyapunov exponent is 1 from  $D_1(\tilde{x}_1) e^{\lambda T}$
- Butterfly speed appears inside  $D_1$  via the dependence on  $\mathbf{M} = \frac{1}{d-1}$
- The butterfly speed defines 'sweet spot', which is a time window where traversability is optimal



Figure 1: arXiv:1409.8180[Roberts, Stanford, Susskind, 14]

#### [Caceres, Misobuchi. Xiao, 18']

The backreaction of the signal inroduces a positive contribution

$$\Delta V 
ightarrow \Delta V_{ ext{back}}, \qquad |\Delta V_{ ext{back}}| \leq |\Delta V|$$

The opening of the wormhole

$$A^U = rac{16\pi G_N}{d-1} \, q^{ ext{tot}}$$

$$\Psi_{\mathcal{O}_R} 
ightarrow e^{i A^U 
ho} \Psi_{\mathcal{O}_R} \,, \quad \delta 
ightarrow \delta_{\mathsf{back}} = \delta + A^U 
ho = rac{16 \pi \, G_N}{d-1} (q+q^{\mathsf{tot}}) \, 
ho$$

$$\Delta V_{\text{back}} = -\frac{4\pi G_N}{d-1} \Delta_{\mathcal{O}} \alpha \ b_{\mathcal{O}}^2 \frac{\text{vol}(S_{d-2})}{(\cosh t_0)^{2\Delta_{\mathcal{O}}+1}} \int_0^\infty d\chi' \frac{\sinh^{d-2}\chi' \Gamma(2\Delta_{\mathcal{O}})}{\left[\cosh\chi' + \frac{4\pi G_N q^{\text{tot}}}{(d-1)\cosh t_0}\right]^{2\Delta_{\mathcal{O}}+1}}$$



**Bound on Information Transfer** 

#### **Bound on Information Transfer**

Briefly review the derivation of information transfer bounds for low dimensional black holes.[Freivogel, Galante, Nikolakopoulou, Rotundo, 19']

Let  $p_V^{\text{tot}}$  be the total momentum of a signal containing  $N_{\text{bits}}$  particles, each one with momentum  $p_V^{\text{each}}$ 

$$N_{
m bits} = rac{p_V^{
m bits}}{p_V^{
m each}}$$

The uncertainty principle states that:

$$ho_V^{
m each} \Delta V_{
m each} \gtrsim 1$$

In order for the signal wave function to pass through the wormhole, we need that

$$\Delta V_{
m each} \leq |\Delta V|$$

which implies that

$$p_V^{\mathsf{each}} \gtrsim rac{1}{\Delta V_{\mathsf{each}}} \geq rac{1}{|\Delta V|}$$

Combining two, we find

$$N_{
m bits} \lesssim p_V^{
m tot} |\Delta V|$$

The phase shift of the collision between the two shocks

$$\delta = S_{\text{classical}} = \frac{1}{2} \int d^{d+1} x \sqrt{-g} \ h_{UU}^{-} T_{+}^{UU} = \frac{4\pi G_N}{r_0^{d-1}} \frac{q_U p_V^{\text{tot}}}{d-1}$$

The probe approximation  $\delta \lesssim 1$  then becomes

$$p_V^{ ext{tot}} \lesssim rac{(d-1)r_0^{d-1}}{4\pi {\sf G}_N {\sf q}_U}$$

The null shift  $\Delta V$  can be written in terms of ANE :

$$\Delta V = \frac{4\pi G_N}{d-1} \mathcal{A}_d^{\infty}(U_0), \quad \mathcal{A}_d^{\infty}(U_0) \equiv \int_{U_0}^{\infty} dU T_{UL}^{-}$$

Finally, we obtain

$$N_{ ext{bits}} \lesssim 
ho_V^{ ext{tot}} |\Delta V| \lesssim \left(rac{(d-1)\,r_0^{d-1}}{4\pi G_N q_U}
ight) rac{4\pi G_N}{d-1} |\mathcal{A}_d^\infty(U_0)| \sim rac{r_0^{d-1}\,h\,K}{(d-a)q_U}$$

with  $a \sim 1.754$ . The above construction only applies for K is less than  $\frac{1}{G_N}$ , [Freivogel, Galante, Nikolakopoulou, Rotundo, 19']

$$N_{
m bits} \lesssim rac{h}{d-a} \, rac{r_0^{d-1}}{G_N} \sim rac{h}{d-a} \, S_{
m BH}$$

# Conclusion

- We study and generalize traversable wormholes in Rindler  $AdS_{d+1}$ .
- We acquire analytic expressions using point splitting method and eikonal approximation method.
- We check the consistency between two results and with the results in the previous works(BTZ case).
- We show information propagates at butterfly speed for localized source
- We acquire the parmeteric bound on information transfer.

Thank you for listening!